The optical mouse for harmonic oscillator experimentation

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The optical mouse is an extremely cost-effective displacement sensor. Here we describe the use of an optical mouse in an experiment to record position as a function of time for a simple mechanical oscillator. The recorded data correspond closely to that expected for damped harmonic oscillations. These measurements can be made without the use of cumbersome attachments. The optical mouse is also advantageous in obviating concern about the integrity of electrical contact points and the deleterious effects of wear and of dirt accumulation on moving parts. © 2005 American Association of Physics Teachers.

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I. INTRODUCTION

An appreciation of mechanical harmonic oscillations is crucial not only for physics students but engineering undergraduates as well. In devising experiments to illustrate mechanical harmonic oscillators, it is desirable to obtain quantitative measurements using electronic data acquisition; in particular with computers. This allows more data points to be collected, which improves accuracy, permits students to devote less time to recording data, and grants them the experience of working with interfaces that are omnipresent in instruments today. Commercial interface hardware can be applied for such purposes, but it tends to be dedicated and expensive. The mechanical mouse offers a convenient method to do this in a cost effective manner, however, due to the nature of operation of the mechanical mouse, some attachments to the roller ball must be made. This makes experimentation with a mechanical mouse cumbersome and reduces the accuracy of the recorded data. Furthermore, there is a need to be concerned with the integrity of the contact points as well as the wearing down of moving parts and the accumulation of dirt. All of these factors conspire to make the experimental setup less robust.

In 1999, Agilent Technologies unveiled the first optical mouse that was immune to the problems of wear and dirt accumulation. Due to the economics of large volume production, the cost of an optical mouse is extremely low. Currently, it is possible to acquire a reasonably good quality unit for as low as US$20. The optical mouse uses a tiny camera to take 1500 pictures or more every second. Able to work on almost any surface, the mouse has a small, red light-emitting diode (LED) that reflects from that surface onto a complementary metal-oxide semiconductor (CMOS) sensor. The CMOS sensor sends each image to a digital signal processor (DSP) for analysis. The DSP, operating at 18 MIPS (million instructions per second) or more, is able to detect patterns in the images and see how those patterns have moved since the previous image. Based on the change in the patterns over a sequence of images, the DSP determines how far the mouse has moved and sends the corresponding coordinates to the computer. The computer moves the cursor on the screen based on the coordinates received from the mouse. This happens hundreds of times each second, making the cursor appear to move very smoothly. The optical mouse has now virtually replaced the mechanical mouse as the standard pointing device for computers.

The optical mouse has been demonstrated to be a practicable optical displacement sensor beyond its use as pointing device. It has been shown to be a cost-effective region-of-interest tracker in microscopy as well as a deformation sensor for viscoelastic materials. These measurements were, however, made with a relatively slowly moving object. More recently, it has been demonstrated that the optical mouse may be applied to measuring vibratory displacements with reasonable accuracy if the vibration frequency was limited to below 10 Hz. Here, we describe an experiment to measure low frequency mechanical oscillations using an optical mouse.

II. MECHANICAL OSCILLATION THEORY IN BRIEF

The equation of motion for a damped oscillator consisting of a mass $m$ attached to a spring with force constant $k$ and a damper with damping coefficient $c$ can be written as

$$m \frac{d^2x}{dt^2} + c \frac{dx}{dt} + kx = 0.$$  (1)

The solution for the displacement $x(t)$ at any time $t$ is given by

$$x = x_0 e^{-ct/m} \cos(\omega_d t + \phi),$$  (2)

where $x_0$ is the initial displacement, $\omega_d$ is the damped angular frequency, and $\phi$ is the phase angle. This treatment assumes that the masses of the spring and damper are insignificant. From Eq. (2), it is clear that the damped frequency of vibration and the damping coefficient are independent of the initial displacement. A verification of this characteristic by experimentation can be posed as a challenge to students.

III. EXPERIMENT

The experiment arrangement used to illustrate mechanical oscillations with an optical mouse (Justy model: UMN-04EMV) is shown in Fig. 1. A mass that hangs from a tensile spring and string is free to oscillate when a small vertical displacement is introduced to the mass. The essential idea is to mount an optical mouse such that it records the vertical movement of the string and thus the mass. For this, a piece of cardboard—which serves as reference surface—is attached via double-sided tape to the string so that its position

corresponds with the recording point of the optical mouse. Once done, it is necessary to place a carefully located guide to prevent accidental out-of-plane motion of the cardboard so that it does not interrupt data recording by the mouse. Attention must be given to ensure that the guide does not create a gap so narrow that the movement of the string is impeded altogether. Prior to carrying out the experiment, students should be advised to carry out a test oscillation to determine a suitable gap.

Once the setup is ready, the connector of the optical mouse is plugged into the computer’s USB port. On the computer, a program11 that polls and stores the vertical position from the optical mouse is activated when the mass is set into oscillatory motion. The beauty of using a USB port mouse lies with allowing it and the original mouse of the computer to be operational in tandem. The original mouse can thus be used to help set the starting position for measurement. Nevertheless, care must be exercised to prevent touching the original mouse once measurement commences as any such movement will result in erroneous measurements. One way to prevent this from happening is to disconnect the original mouse during measurement. The recorded data can be transferred to a spreadsheet program such as Microsoft Excel to perform the analysis.

In the experiment, the mass used was 0.25 kg. From a simple extension test, the spring constant was determined to be 81.75 N/m. The natural frequency of vibration was hence approximately 2.87 Hz when calculated using the simple relation $f_n = \sqrt{\frac{k}{4\pi^2m}}$ for an undamped oscillator. For the experiment, five sets of data with varying initial amplitude were collected for comparison. In a calibration test, one pixel recorded on the optical mouse was found to correspond to 0.0625 mm.

IV. RESULTS AND DISCUSSION

Figure 2 shows a typical position versus time curve obtained using this apparatus. It can be seen that the data points depict the expected trend of a damped mechanical oscillation very well. It was understood from theory that the harmonic frequency should be independent of the initial amplitude applied. Hence, it is desirable to determine the (damped) harmonic frequency of each data set. One method for doing this is to approximate the period from the position versus time curve; however, a far more accurate means of doing this is to determine it from a computed power spectrum using the position versus time data. Figure 3 shows a plot of the power spectrum of the curve shown in Fig. 2 in the positive frequency range. The frequency of vibration can be determined from the location of the peak in the frequency spectrum.

The results from the five sets of data collected are summarized in Table I. The relatively low deviation of the harmonic frequency from the mean (2.4%) confirms the independence of harmonic frequency on the initial amplitude. The difference between mean measured (damped) frequency, 2.119 Hz, and the natural (undamped) frequency of the system, 2.87 Hz, provides a useful opportunity for discussion, notably, of the significance of the damped frequency value being lower than the natural frequency, as well as the relative closeness of the two values.

It is also expected from theory that the damping coefficient should be independent of the initial amplitude introduced. A relatively simple method of characterizing the damping is to generate a function of the form $A + B \exp(-Ct)$ to represent the upper-bound curve, and another function $A - B \exp(-Ct)$ to represent the lower-bound curve on the position–time graph. The idea is then to modify the upper and lower bound curves so that they match closely the am-
Table I. Harmonic frequency, damping coefficient, and initial amplitude determined from the five sets of position versus time data.

<table>
<thead>
<tr>
<th>Data set</th>
<th>Damped frequency (Hz)</th>
<th>Damping coefficient (Ns/m×10^{-5})</th>
<th>Initial amplitude (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.116</td>
<td>10.0</td>
<td>5.625</td>
</tr>
<tr>
<td>2</td>
<td>2.038</td>
<td>7.5</td>
<td>8.125</td>
</tr>
<tr>
<td>3</td>
<td>2.152</td>
<td>9.0</td>
<td>5.938</td>
</tr>
<tr>
<td>4</td>
<td>2.116</td>
<td>8.5</td>
<td>6.563</td>
</tr>
<tr>
<td>5</td>
<td>2.171</td>
<td>7.0</td>
<td>5.000</td>
</tr>
<tr>
<td>mean ($)</td>
<td>2.119</td>
<td>8.4</td>
<td>...</td>
</tr>
<tr>
<td>standard deviation ($\sigma$)</td>
<td>0.051</td>
<td>1.2</td>
<td>...</td>
</tr>
<tr>
<td>$\sigma/\bar{f}$</td>
<td>0.024</td>
<td>0.14</td>
<td>...</td>
</tr>
</tbody>
</table>

The results from the five sets of data are summarized in Table I. The mean value of the damping coefficient was $8.4 \times 10^{-5}$ Ns/m, which indicates a low degree of damping present in the system. The relatively small deviation of the damping constant values about the mean (14.2%) confirms the independence of the damping coefficient from the initial amplitude introduced. It should be noted that this value is higher than is the case with the standard deviation of the harmonic frequency (2.4%) about the mean. This may be due to the relatively inaccurate method by which the damping coefficient is approximated from the position-time curve. Table I also gives approximate values of the initial amplitude for each data set. The values obtained confirm the correct manner in which the random variable for the experiment is introduced.

V. CONCLUSIONS

This work presents an experiment using an optical mouse to record mechanical oscillations. The favorable attributes of the optical mouse allow accurate measurements to be obtained and for the setup to be robust. In a simple test, the experiments produced data that showed good correspondence with the expected independence of harmonic frequency and damping coefficient on initial amplitude. The positive results with the optical mouse obtained here portend its applicability in other mechanical oscillator experiments that involve oscillations with more degrees of freedom.


11http://twng.eng.nus.edu.sg/course/ResearchOM/mousetrackerdynamic.html

The literature for teaching geometrical optics at the high school or college level is filled with ray diagrams.1–4 These diagrams are of immense value in helping students understand the principles of optics. Often these diagrams depict parallel rays incident on optical elements that then converge the rays to a focal point; however, in laboratory activities at the introductory level, students rarely have a way of visualizing rays in three dimensions. To overcome this difficulty, we have devised a means of selecting parallel rays that can be used to measure the focal length of mirrors and lenses. In addition, students can map the dependence of the focal length of an optical element as a function of the distance of an aperture from the principal axis. Here we describe, as an example, a scheme for selecting principal rays when using a concave mirror.

We use an incandescent lamp in a darkened room as a light source. The lamp is placed five to ten focal lengths from the mirror, which is far enough to justify a parallel-ray treatment yet a manageable distance (about 5 m in the case described here) in the laboratory. The crux of the activity is to cover the entire mirror with a mask having small cut-out apertures arranged symmetrically about the principal axis of the mirror. A typical mask, as shown in Fig. 1, is created in CorelDRAW,5 but other graphic programs can be used as well. For readers without access to a graphics program, masks are easily made using a compass and straight edge.

The aperture shape is unimportant, and it is not necessary that all apertures have the same shape. The particular mask shown in Fig. 1 consists of an array of small equilateral triangles. To create a particular mask, students use a razor-blade knife to cut out only those apertures that provide the rays they wish to use. Triangles were chosen as an example

Masks for selecting parallel rays for geometrical optics activities

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